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PROJECT PROPOSAL: TRIPLE PRODUCT L-FUNCTION

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In number theory it is often the case that understanding the behavior of certain so-called *L-functions* at specific values leads to arithmetic results. For example, a celebrated result of Dirichlet states that for m and n relatively prime integers, there are infinitely many prime numbers among the sequence

$$m, m+n, m+2n, m+3n, m+4n, \ldots.$$

This fact can be deduced from knowing that for every non-trivial Dirichlet character χ modulo n, the L-function $L(\chi, s)$ is holomorphic at $s = 1$, whereas the Riemann zeta function $\zeta(s)$ has a (simple) pole at $s = 1$.

In modern parlance, the character χ can be interpreted as an *automorphic representation* of GL_1. An automorphic representation π of GL_2 corresponds to another important object for number theorists: namely, a modular form. In slightly more generality, given a quaternion algebra B and three autormorphic representations π_1, π_2 and π_3 of B, one can form the *triple product* $\Pi := \pi_1 \otimes \pi_2 \otimes \pi_3$. An important formula of Ichino roughly states that for $\varphi \in \Pi$,

$$\int_{[B]} \varphi(b)\, db = C \cdot L(\Pi, \tfrac{1}{2}) \cdot \prod_{v \in S} I_v(\varphi),$$

where C is constant and S is a finite set (which depends on Π and φ)[1]. For number theoretic applications, it is usually necessary to understand well the *local factors* I_v which have been studied in many—but not all—cases.

In this project we intend to consider analogues of the local factors in the case of groups defined over finite fields. This will require learning about and applying the theory of representations of finite groups. It could also involve enlisting the help of a computer to do calculations and collect data. Depending on progress made, this could lead to studying the representation theory of groups defined over real and/or p-adic numbers. We may also explore possible applications of our formulas.

Interested students should have taken at least one semester of abstract algebra, meaning that they have studied finite groups and some field theory. Programming experience is preferred. Having taken courses in Number Theory, Representation Theory, Complex and/or Real Analysis would be a plus.

[1]Don't worry if all or most of this paragraph is gibberish. Learning what it means is part of the project.