Applied Combinatorics

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Answers to Selected Exercises[1]

Chapter 13

Section 13.1.
1(a). Add edges in the order $\{b,e\},\{d,e\},\{a,e\},\{c,d\}$;
1(b). Add edges in the order $\{a,b\},\{b,c\},\{d,e\},\{e,f\},\{g,h\},\{h,i\},\{a,d\},\{d,g\}$;
1(c). Add edges in the order $\{c,g\},\{a,b\},\{a,d\},\{f,g\},\{d,f\},\{e,f\}$ where ties were broken arbitrarily;
2(a). add edges $\{a,e\},\{b,e\},\{d,e\},\{c,d\}$;
2(b). Add edges in the order $\{a,b\},\{b,c\},\{a,d\},\{d,e\},\{e,f\},\{d,g\},\{g,h\},\{h,i\}$;
2(c). Add edges in the order $\{a,b\},\{a,d\},\{d,f\},\{f,g\},\{c,g\},\{e,f\}$;
3(a). terminate with message disconnected; T ends up with 5 edges;
3(b). terminate with message disconnected; T ends up with 7 edges;
3(c). terminate with message disconnected; T ends up with 6 edges;
5. vat pairs: $\{7,8\},\{6,7\},\{1,5\},\{4,5\},\{5,8\},\{2,3\},\{3,8\}$;
6. component pairs: $\{1,4\},\{2,6\},\{3,4\},\{1,6\},\{5,6\}$;
9. (a): edges $\{b,c\},\{c,e\},\{c,d\},\{a,b\}$; (b): edges $\{f,i\},\{e,h\},\{d,g\},\{c,f\},\{b,e\},\{a,d\},\{g,h\},\{h,i\}$; (c): edges $\{d,e\},\{c,d\},\{a,e\},\{b,c\},\{e,f\},\{d,g\}$;
11. yes;
12(a). edges $\{a,e\},\{a,b\},\{c,d\},\{d,e\}$;
12(b). edges $\{c,e\},\{a,b\},\{c,d\},\{b,d\}$;
13. in Step 1 of Kruskal's Algorithm, set $T = \{$ the set of edges specified as having to belong to the spanning tree $\}$;
16(a). for network (c): G' has edges $\{a,b\},\{c,g\},\{a,d\},\{f,g\},\{d,f\}$; in the next iteration we add $\{e,f\}$ and obtain a minimum spanning tree.

Section 13.2.
3(a). We successively add to $W: a,b,d,c,e,z$, obtaining the path a,d,e,z;
3(b). We successively add to $W: a,c,b,f,d(\text{or } e),e(\text{or } d),z$, obtaining the path a,b,e,z;
3(c). We successively add to $W: a,b,c,e,d,z$, obtaining the path a,c,e,z;
3(d). We successively add to $W: a,b,e,h,c,f,i,j,g,d,z$, obtaining the path a,h,i,j,z;
4. the path a,b,d,g,i;

[1] More solutions to come. Comments/Corrections would be appreciated and should be sent to: Barry Tesman (tesman@dickinson.edu) or Fred Roberts (froberts@dimacs.rutgers.edu).

5. grind, weigh, polish, inspect;
6. buy in year 1, sell in year 3, buy in year 3 and then sell in year 6;
10. $(n-2)!$;
12(b). $1, 4, 6, 7$;
12(c). first line: words $1, 2, 3$; second line: words $4, 5$; third (last) line: word 6;
14. it will take the boatman seven trips across the river. First the boatman takes the goat across the river. He then goes back and takes the wolf across. He drops the wolf off, but at the same time puts the goat back into the boat. He takes the goat back across the river. The boatman drops the goat off, but at the same time puts the cabbage in the boat. He takes the cabbage across and then goes back and gets the goat;
15. 7: draw a digraph D where
$V(D) = \{x = (x_1, x_2, x_3) : x_i =$ number of gallons in jug $i\}$, and
$A(D) = \{(x, y) : y$ is attainable from x by pouring one jug into another$\}$ and find a shortest path from (8, 0, 0) to (4, 4, 0);
19. no;
21. let $\bar{d}(i, j)$ be the distance from i to j and let $\mathbf{A}$ = adjacency matrix; then $\bar{d}(i, j)$ is the smallest k such that the i, j entry of $\mathbf{A}^k$ is nonzero.

Section 13.3.
1. for network (c): **(a)** feasible, **(b)** value 4;
2. (a): 8;
3. (a): $s_{sa} = 3, s_{sb} = 0, s_{ba} = 1, s_{at} = 0, s_{bt} = 2$;
4(a). yes;
5(b). no;
6. (a): augmenting chain s, a, b, t;
17(b). for graph (a), let the flow be 1 on arcs $(s, a), (s, b), (s, d), (a\alpha), (b, \beta), (d, \gamma), (\alpha, t), (\beta, t), (\gamma, t)$, and 0 otherwise; the matching is $\{a, \alpha\}, \{b, \beta\}, \{d, \gamma\}$;
17(c). for graph (a), the (s, t) cut $S = \{s, c, d, \alpha, \beta, \gamma\}, T = \{t, a, b\}$ has corresponding covering $\{a, b, \alpha, \beta, \gamma\}$;
20. (a): $x_{sa} = 3, x_{sb} = 2, x_{at} = 1, x_{ab} = 2, x_{bt} = 4, x_{sc} = 3, x_{ct} = 3, \text{rest} = 0$;
22. (a): $F(0011) = 1, F(0010) = 0$, and so on;
23. (a): $x_{ad} = 11, x_{bd} = 1, x_{df} = 12, x_{ce} = 5, x_{eh} = 5, \text{rest} = 0$;
36. argue by induction on the length of C that if C is flow-augmenting, it contains a simple flow-augmenting chain.

Section 13.4.
1. (a):
$x_{sa} = x_{at} = 2, x_{sb} = x_{bt} = 1, x_{ba} = 0, \text{or } x_{sa} = x_{at} = 1, x_{sb} = x_{bt} = 2, x_{ba} = 0$;
10(a). (13.13) and (13.14) give us
$\sum_{i=1}^{n} \sum_{j=1}^{m} x_{ij} \le \sum_{i=1}^{n} a_i < \sum_{j=1}^{m} b_j \le \sum_{j=1}^{m} \sum_{i=1}^{n} x_{ij}$;
11(a). In (a), such a flow has $x_{sa} = 1, x_{at} = 2, x_{sb} = 2, x_{ba} = 1, x_{bt} = 1$ and negative cost augmenting circuit is t, a, b, t.