# Applied Combinatorics

## by Fred S. Roberts and Barry Tesman

### Answers to Selected Exercises[1]

### Chapter 1

**1**. $\begin{array}{|cccc|} \hline 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \\ \hline \end{array}$;

**2**. $\begin{array}{|ccc|} \hline 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \\ \hline \end{array}$; $\begin{array}{|ccc|} \hline 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \\ \hline \end{array}$; $\begin{array}{|ccc|} \hline 1 & 3 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \\ \hline \end{array}$; $\begin{array}{|ccc|} \hline 1 & 3 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \\ \hline \end{array}$; $\begin{array}{|ccc|} \hline 2 & 1 & 3 \\ 1 & 3 & 2 \\ 3 & 2 & 1 \\ \hline \end{array}$; $\begin{array}{|ccc|} \hline 2 & 1 & 3 \\ 3 & 2 & 1 \\ 1 & 3 & 2 \\ \hline \end{array}$;
$\begin{array}{|ccc|} \hline 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \\ \hline \end{array}$; $\begin{array}{|ccc|} \hline 2 & 3 & 1 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \\ \hline \end{array}$; $\begin{array}{|ccc|} \hline 3 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \\ \hline \end{array}$; $\begin{array}{|ccc|} \hline 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \\ \hline \end{array}$; $\begin{array}{|ccc|} \hline 3 & 2 & 1 \\ 1 & 3 & 2 \\ 2 & 1 & 3 \\ \hline \end{array}$; $\begin{array}{|ccc|} \hline 3 & 2 & 1 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \\ \hline \end{array}$;

**3**. Let row 1 be: $1\ 2\ \cdots\ n$. Row 2 is gotten by taking the first element (1) of row 1 and moving it to the end of the row. Row 3 is gotten from row 2 by taking the first element (2) of row 2 and moving it to the end of the row. Continue until you have $n$ rows.

**4(a)**. $\begin{array}{|ccc|} \hline 3 & 2 & 1 \\ 1 & 3 & 2 \\ 2 & 1 & 3 \\ \hline \end{array}$; $\begin{array}{|ccc|} \hline a & b & c \\ b & c & a \\ c & a & b \\ \hline \end{array}$;

**4(b)**. $\begin{array}{|cccc|} \hline 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \\ 2 & 1 & 4 & 3 \\ 3 & 4 & 1 & 2 \\ \hline \end{array}$; $\begin{array}{|cccc|} \hline a & b & c & d \\ c & d & a & b \\ d & c & b & a \\ b & a & d & c \\ \hline \end{array}$;

**5**. 0, 1, 00, 01, 10, 11, 000, 001, 010, 100, 011, 101, 110, 111;

[1]More solutions to come. Comments/Corrections would be appreciated and should be sent to: Barry Tesman (tesman@dickinson.edu) or Fred Roberts (froberts@dimacs.rutgers.edu).

**6**. See problem 5 and 0000, 0001, 0010, 0100, 1000, 0011, 0101, 0110, 1001, 1010, 1100, 0111, 1011, 1101, 1110, 1111;

**7(a)**. no - there are only 12 such strings;

**7(b)**. yes - there are 27 such strings;

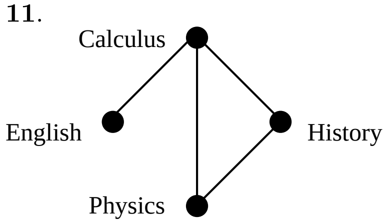
**8**. There are 84 such strings: there are 4 of length one; there are 16 of length two; there are 64 of length three;

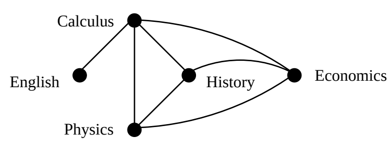
**9**. LLLL, LLLS, LLSL, LSLL, SLLL, LLSS, LSLS, SLLS, LSSL, SLSL, SSLL, LSSS, SLSS, SSLS, SSSL, SSSS;

**10**. Our conclusion would not change significantly. There are roughly $3.15 \times 10^7$ seconds per year and 100 billion equals $10^{11}$. So, $3.15 \times 10^7 \times 10^{11}$ or $3.15 \times 10^{18}$ networks could be analyzed in a year. Then the number of years it would take to check $6 \times 10^{33}$ networks is

$$\frac{6 \times 10^{33}}{3.15 \times 10^{18}} \approx 1.9 \times 10^{15};$$

**11**.

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**12(a)**. No assignment exists. Each of Calculus, History, and Physics must get a different exam time since they overlap with one another;

**12(b)**. time 1: English and Physics; time 2: Calculus; time 3: History;

**12(c)**. No assignment exists. Each of Calculus, History, Physics, and Economics must get a different exam time since they overlap with one another;

**12(d)**. time 1: English and Physics; time 2: Calculus; time 3: History; time 4: Economics;

**13**. If Economics is Wednesday and Transportation is Tuesday then both Housing and Health must be Thursday - this is not possible. Or, if Economics is Wednesday and Transportation is Thursday then both Housing and Health must be Tuesday - again this is not possible.

**14(a)**. If English must be Thur. AM, then Calculus must be Wed. AM. But then History and Physics must be Tues. AM - this is not possible.

**14(b)**. Wed. AM: English; Tues. AM: Calculus; Wed. AM: History; Thur. AM: Physics; Mon. AM: Economics.

**15**. Two of the four instructors can get their first choice. Assign a different morning to each of the Calculus, History, and Physics exam times. Then assign Tuesday morning to English.